Liquidity risk in Fixed Income Markets

Valuation adjustment, risk assessment, stress testing and portfolio construction

Research paper #3
Natixis Asset Management
Fixed income investment division

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EXECUTIVE SUMMARY

Market risk and liquidity risk are by far the main sources of uncertainty affecting a portfolio’s future P&L. While the former can be explained by uncertainty regarding price fluctuations, the latter is incurred when trading assets. Illiquidity increases with the size of the position. It occurs over some short term but vanishes over a longer horizon. Typically a security held to maturity has no liquidity cost. However, unlike other risk factors, liquidity risk cannot be diversified. For example, one cannot offset a given level of liquidity “exposure” by going short an illiquid security. More generally, no known liquidity-based derivatives could hedge this particular risk. Indeed, in stressed markets, bid rather than mid prices prevail. In this paper, after a brief survey of the financial theory on liquidity risk, we examine its main characteristics, its measures and drivers as well as its preeminent role in the development of crisis and the burst of bubbles. Liquidity thus appears as a risk factor that signals returns ex ante and explains performance ex post, at least partially. According to the existing literature, liquidity risk premium would be around 0.6% for investment grade bonds and 1.5% for speculative bonds. Extending equity liquidity measures to bond market is not henceforward as they depend on bond’s intrinsic characteristics. Unlike stocks, bonds redeem. The liquidity of 10 years bond is not the same as 3 months one even for the same issuer. A traded volume based liquidity measure can be quite misleading as a traded bond is not necessary a liquid one e.g. forced selling and falling angels and vice versa bonds not traded are not necessary illiquid. We use Barclays’ Liquidity Cost Scores (LCS) as a measure for liquidity in credit markets. Liquidity cost typically falls as issue size and volume increase, option adjusted spread (OAS), duration times spread (DTS) and age decreases. Notably, we find that omitting liquidity risk can underestimate credit portfolio VaR (99%) by up to 22%. We also discuss appropriate techniques for recovering liquidity premiums and extracting fair prices, depending on the degree of market liquidity. We broaden our analysis to present a general framework that encompasses both market and liquidity risk, and measure the impact of both at the security and portfolio levels. In short, we highlight the importance of explicitly taking liquidity into account in portfolio construction, and propose a methodology to do so.
Until recently, the market paradigm encapsulated in the financial theory did not explicitly account for liquidity risk, and rather assumed that investors could buy and sell significant position sizes without affecting market prices. As a consequence, securities are priced, and their risk measured, at the mid price irrespective of any friction stemming from liquidity. However, in nervous markets, liquidity recedes and the bid price becomes the only relevant value. Such circumstances call into question the notion of mark to market, as it no longer reflects a “fair” price.

Financial history is replete with liquidity crises. During the 1998 LTCM crisis, hedge fund positions had grown so large that it was impossible to liquidate them without significant price impact. More recently, in the 2007-2008 financial crisis, widespread liquidity shortage forced banks to reduce exposures by liquidating assets. Many asset managers became forced sellers to meet outflows and margin calls. Prices dropped as liquidity melted.

The over the counter (OTC) nature of fixed income markets makes tracking liquidity risk much more challenging than in equity markets which are more centralized and use a single price. For example, whereas the 2007-2008 crisis forced European Fixed income markets to shut down temporarily, the “Trace” system (unavailable in Europe) enabled US market operators to track which bonds had traded and at what price. This enabled US markets to remain open despite minimal volume. This led to Barclays transposing their Liquidity Cost Score (LCS) methodology in Europe mid 2010 following its existence in the US since October 2009.

Following the financial crisis, policymakers and regulators have sought to impose tougher rules and standards on banks to prevent future systemic crises. Basel III introduced new liquidity standards, namely new liquidity ratios and higher-quality liquid assets. Notwithstanding, the very issue such standards seek to address can be taken by some market participants as a damper to liquidity and thus dissuasive to active management. Indeed, in Europe and the US alike, higher restrictions on RWAs impose higher costs for market making activities. The subsequently diminished risk “envelope” for such activities compromises available market liquidity for secondary trading. The direct consequence, illustrated below, is that US broker/dealer corporate bond inventories have substantially diminished since 2007.

One could be frightened up when discovering the evolution of dealer’s inventory of corporate bonds in the US: it has impressively diminished since 2007! Although the corporate bond market has doubled in size since 2001, the available inventories remain largely unchanged!

RWA: Risk Weighted Assets.
Amihud et al. (1991) posited that a change in investor perception of liquidity risk would push them to price securities at lower levels which could result in a crash akin to October 1987. The root of liquidity risk lies in information asymmetries and the existence of incomplete markets which lead to adverse selection and moral hazard situations. It follows that in order to reduce systematic liquidity risk, transparency and information flow should be enhanced. Nonetheless, this could be a very costly strategy that requires a long time before being fully operational.

DEFINING AND MEASURING LIQUIDITY RISK

I. LIQUIDITY: AN HETEROGENEOUS CONCEPT

When talking about liquidity, some caution is required as the nature of risk is not the same when analyzed on a global macro economical level, on a corporate level or on a specific asset level.

From a broad economical perspective, liquidity risk is the ability of economic agents to exchange their wealth into goods. Liquidity is thus a flow notion (in opposition to the notion of stock). Illiquidity arises in case of inability of exchanging. When money lacks in the overall economy, transactions and thus activity slow down. Banks finance the economy through lending operations. By doing so, they boost the economical activity by supporting investment and consumption. However, banks have to meet some legal obligations in terms of reserve requirements which create a global deficit in liquidity. Banks rely on central bank to offset this liquidity deficit and to get refinanced.

Central bank can improve (or sterilize) global liquidity of the economy by acting on the monetary base through open markets operations in a way to keep the interbank lending rates close to its target rate. The latter is a tool to monitor the overall liquidity with respect to certain objectives e.g. inflation and/or growth.

From a theoretical perspective, asymmetrical information hinders exchanges leading to illiquidity and market incompleteness. Central bank plays an important role in managing a liquidity crisis. “It can act as an immediate but temporary buffer to liquidity shocks, thereby allowing time for supervision and regulation to confront the causes of liquidity risk.”

From a corporate perspective, liquidity denotes the solvability of the firm e.g. the net liquidity of assets and liabilities. When assets’ cash flows are no longer sufficient to cover liabilities, the firm is facing a credit event that may lead to bankruptcy. Default risk varies from one country to another according to the economical cycle. It depends also on the sector and other factors.

From a market perspective, investors are concerned with asset liquidity issues generally defined as the ease of trading the asset. In the following sections we will focus our attention on this specific risk.

II. ASSET LIQUIDITY RISK

Liquidity is an elusive notion, not only because it applies to different levels of the economy with complex linkages but also because it is a multi dimension concept difficult to capture in a single measure and thus to model. Even on a single asset level, liquidity appears as a heterogeneous concept.
The term liquidity risk has a negative connotation as it occasions costs and losses. The cost of liquidity when trading an asset is typically captured by the bid-ask spread. This spread, at least from the market maker perspective, can be broken into two components: the effective spread which compensates market maker for insuring the liquidity of the market and corresponds to the technology and inventory costs incurred by the market maker, the information spread that compensate market makers for potential loses they may incur when taking an uninformed bet. Since they cannot distinguish informed from uninformed bets, they might well be “the opposite side” of an informed order in their role of insuring market liquidity.

Amihud and Mendelson (2006) split liquidity costs into three components:

- **Direct trading costs**: deterministic transaction costs encompassing brokerage commissions, transaction taxes and exchange fees.
- **Price impact costs** correspond to the difference between the executed price and the mid one. It is limited to (half) the bid-ask spread for small orders but can exceed this spread for higher positions. When trading a small position, a single counterparty is sufficient to execute the order at the best price. As the size of the position increases, many counterparties are required to absorb the order, each with different beliefs about the fair value of the asset, which leads to a lower price.
- **Search and delay costs** incurred when traders delay the execution and search for a better execution price than the one “displayed” by the bid-ask spread. By doing so, traders undertake the risk of seeing the market move by the time they decide to execute their order. This tradeoff between price impact costs and seeing the market move is particularly relevant for block orders.

By denoting a liquid market as one in which every agent can buy and sell at any time a large quantity rapidly at low cost, Harris (1990) distinguishes four interrelated dimensions for liquidity:

- **Width**, that measures the cost incurred by a round trip transaction e.g. by instantaneously buying and selling a position. The costs incurred will thus correspond to the price impact costs and direct trading costs.
- **Depth**, which is the number of shares that can be traded at a given price without incurring additional costs above the bid-ask spread. According to Bangia et al. (1999), up to the quote spread, liquidity costs are exogenous as the market is able of absorbing the position. The quote applies to all market participants irrespective to their characteristics. For higher position, liquidity costs are assumed endogenous as they are supposed to be specific to the individual trade position. Chart 1 shows bid-ask spread as a function of the quote depth.

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![Chart 1: Effect of position size on liquidation value](chart.png)

Above a cut-off size, illiquidity becomes endogenous and its burden higher
Source: Bangia et al. (1999)

5 Negative basis strategy, e.g. when a CDS is lower than the underlying bond’s spread, is a typical counterexample where liquidity offers very attractive risk reward opportunity for investors.

6 The distinction between exogenous and endogenous liquidity is challenged by Stange and Kaiserer (2009) who argue that whole price impact curve is exogenous because it is determined by the market.
Liquidity risk in Fixed Income Markets

Immediacy, which captures how quickly positions can be traded and corresponds to time between placing the order and its settlement.

Resiliency that indicates the ability of the market to absorb random shocks e.g. uninformative orders.

Stange and Kaserer (2009) argue that liquidity is a continuous characteristic and distinguish 4 degrees of liquidity (reproduced in Chart 2) according to the liquidity costs they occur:

- Costless trading when any position can be traded without any cost.
- Continuous trading when most of the orders are executed at a certain cost.
- Interrupted trading when some orders are executed from time to time.
- No trading when the market is completely illiquid, prices are not available and should be recovered by suitable techniques.

Those degrees of liquidity depend on the asset type, the size of the position and the liquidation horizon. Exchanging cash is a costless trading as it does not require any value adjustment. Exotic securities are traded interruptedly while some structured credit products like CDO and ABS were typical illiquid assets during the 2008 financial crisis. Illiquidity increases with the size of the position as explained before. It occurs over some short term but vanishes over a longer horizon. Typically a security held to maturity has no liquidity cost.

This differentiation will be useful later on for selecting an appropriate method to incorporate liquidity adjustment and determine the fair value prices according to the degree of liquidity of the market.

III. MEASURING LIQUIDITY RISK IN FIXED INCOME MARKETS

Almgren and Chriss (2000) stressed the importance of distinguishing temporary price impact from permanent one when determining the optimal execution of portfolio transactions in a dynamical liquidity framework. Temporary price impact is due to transitory imbalances in supply and demand caused by one’s trading and leading to an actual price lower than the equilibrium/mid one. It vanishes rapidly according to market’s resiliency. Permanent price impact entails a change in the equilibrium/mid price caused by one’s informed trade at least until the end of the liquidation horizon. The trade contains “real” information that affects the equilibrium price.

III. MEASURING LIQUIDITY RISK IN FIXED INCOME MARKETS

As mentioned in Chacko (2005), existing studies have focused on US equities due to data limitation and the sparse nature of bond market. Extending equity liquidity measures to bond market is not henceforward as they depend on
bonds’ intrinsic characteristics. Unlike stocks, bonds redeem. The liquidity of a 10-year bond is not the same as a 3-month one even for the same issuer. A traded-volume-based liquidity measure can be quite misleading as a traded bond is not necessary a liquid one e.g. forced selling and falling angels and vice versa bonds not traded are not necessary illiquid.

At some point, one has to come back to reality: what liquidity can be expected from typical issuance sizes of €M 500 (corporate benchmark size) vs. dozens of billions issued for a typical government bond benchmark?
What liquidity can be expected when such issuances are 3 to 5 times oversubscribed and kept until maturity in buy-and-hold portfolios (Life insurers have become the dominant player in corporate bonds in the last 7 years): only 20-30% of free float is available. It cannot be improved without an active Repo market on such assets.
Dastidar and Phelps (2009) introduced the liquidity cost score (LCS) to measure bond level liquidity. LCS are computed by Barclays Capital© on a monthly basis over a wide range of fixed income securities (IG, HY, Covered, MBS) and regions (US, Euro).
A bond’s LCS, represents the round-trip cost, as a percent of the bond’s price, of immediately executing a standard institutional transaction. So according to this definition, a lower LCS value denotes better liquidity. More formally, LCS is computed as follows:

\[
LCS = \begin{cases} 
(Bid - Ask)\text{Spread} \times OASD & \text{if bond is spread - quoted} \\
\frac{Ask \text{ Price} - Bid \text{ Price}}{Bid \text{ Price}} & \text{if bond is price - quoted.} 
\end{cases}
\]

For non-quoted bonds, LCS is estimated given bonds characteristics. Liquidity cost typically falls as issue size increases, volume increases, option adjusted spread (OAS) decreases, duration times spread (DTS) decreases, or age decreases. (see boxed text)

Given the Barclays Euro Aggregate Corp Index (BEAC) constituents observed from July 2012 till June 2013, we noticed that liquidity on a security by security level improves with the issue size and the price and that higher LCS were concomitant to higher maturity issues, higher yield to maturity (YTM), Libor option adjusted spread (LOAS), duration and DTS as reproduced in the Table 1.

These findings suggest that liquidity risk is priced by the credit market, at least partially.
On Chart 3, we can check that LCS varies among and within sectors and seniority. Subordinated securities have a higher LCS. These findings are consistent with the high correlation of LCS with DTS.

"A corporate bond is only liquid during its life time when it is issued in primary markets!" (trader’s joke)

The sector partition is the one used by the credit investment team in NAM’s Fixed Income department.
One can notice that, on average, the higher the Beta of the sector (i.e. Subordinated Insurance), the larger the LCS. It reached a paroxystic level during 2008 end and the beginning of 2009 (crisis times) when perpetual bonds reached distressed prices ca 20-30% of par: at that time it was not unfrequent to suffer from 4%-5% bid-ask spread!

2 THE LIQUIDITY FACTOR

I. WHAT FACTORS DRIVE LIQUIDITY?

Theory and empirical studies suggest that liquidity risk is priced by the market e.g. higher illiquidity implies lower prices and higher expected returns. However, unlike other risk factors liquidity risk cannot be diversified. One cannot offset a liquidity exposure by going short an illiquid security. More generally there is no liquidity based derivatives to hedge this particular risk.

However, liquidity can be managed. Portfolio managers for instance may choose their liquidation policy with respect to the illiquidity cost profile of underlying securities keeping illiquid assets for a longer time and trading more frequently liquid securities. It is not a free lunch, and liquidity management may cause some performance drawdown.
Every active portfolio manager has realized that alpha generation was excessively impacted nor to say shrunk to a certain degree by large bid ask spreads: being active is required in order to beat any benchmarks but it gets impossible to reach this aim if one cannot implement its active strategies.

The trade off between performance, risk and liquidity is a key element in managing public funds subject to inflows and outflows of cash. Besides the usefulness of keeping a liquid basket to face outflows without burdening a prohibitive liquidity cost, it is convenient to monitor overall liquidity of the market as it seems to be related to other market factors like implied volatility, Libor-overnight indexed swap (LOIS) spread, etc. However the correlation is far from being perfect (61% with V2X and 79% with the difference between LOIS over the period June 2010 - June 2013). Moreover a hedge using V2X may be difficult to implement for Fixed Income managers.

We can check on Chart 4 that correlation between LCS and V2X and LOIS reaches its maximum when lagging the latter up to 4-7 weeks, suggesting that V2X and LOIS impact on liquidity attains its maximum with some delay. However advanced causality tests, based on Independent Component Analysis for example, do not corroborate the leading predictive capacity of V2X and LOIS on LCS. Moreover, we can check on Chart 5 and on Chart 6 the difficulty of estimating a robust relationship with or without lags.

**Table 2: Summary table for monitoring liquidity costs in PACT**

<table>
<thead>
<tr>
<th>Date</th>
<th>Benchmark LCS*</th>
<th>Portfolio LCS</th>
<th>Break even relative spread movement</th>
<th>Turnover</th>
<th>Turnover cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>October 13</td>
<td>42bp</td>
<td>50bp</td>
<td>-1.41%</td>
<td>7.2%</td>
<td>2.8bp</td>
</tr>
</tbody>
</table>

*The benchmark is the Barclays Euro Aggregate Corporate.

**Chart 4: Correlation between LCS and lagged V2X and LOIS**

Correlation reaches its max with a 4-7 weeks delay

Source: Natixis Asset Management

Correlation between LCS and lagged market variables

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Liquidity risk is closely monitored by the Credit investment team at Natixis Asset Management’s Fixed Income department. Funds are kept inside a range of leeway with respect to a model portfolio. Portfolio managers express their views regarding future relative spread movements for both top down (directional and sector) and bottom up (security selection) strategies. Mixing all of these inputs in a single framework is the key for generating a robust and repeatable alpha while addressing liquidity costs incurred by active trading. A dedicated proprietary tool, Portfolio Allocation and Construction Tool (PACT), has been developed to address this issue.

When building the next period model portfolio using PACT, liquidity risk ranks high among other risk factors. The expected excess returns stemming from portfolio managers’ views are put in balance with the cost of implementing these views. Break even relative spreads are derived for each aggregate bucket. If the expected gain of implementing a view is offset by the liquidity cost generated by turnover, the strategy is challenged. In general, the credit model portfolio construction is conducted under the objective of minimising liquidity costs and thus turnover on an issue by issue level.

For instance, in order to rebalance the credit model portfolio as of October 2013 a minimum turnover of 7.2% is required. This turnover would have cost less than 3bp. Portfolio managers would have been better off not adjusting their exposure if spreads were to tighten less than 1.4% over the investment horizon as we can check on Table 2.
II. THE LIQUIDITY PREMIUM

Amihud (2002) shows that expected market illiquidity positively affects ex ante stock excess return, suggesting that expected stock excess return partly represents an illiquidity premium. This complements the cross-sectional positive return–illiquidity relationship. This increasing relation between illiquidity and return still holds after controlling for risk and some other characteristics.

Amihud and Mendelson (1986, 1988) show that the relation between asset returns and illiquidity is upward sloping and concave. Returns increase less for highly illiquid assets. This pattern holds for stocks and bonds as advocated in Amihud and Mendelson (1991). For corporate bonds, Chen et al. (2007) found that illiquidity costs increase as rating deteriorates and that change in bonds illiquidity costs lead to changes in bonds yield.

Jong and Driessen (2012) show that corporate bond returns have significant exposures to liquidity fluctuations. Liquidity risk premium would be around 0.6% for investment grade bonds and 1.5% for speculative bonds. Further, this liquidity risk is a priced factor for the expected returns. Since liquidity is a risk factor, investors ask for higher return to get compensated for bearing this risk.

Looking at LCS dynamics for the Barclays Euro Aggregate Corp Index (BEAC) sectors, we also found a positive dependency with YTM especially when YTM are tightening e.g. since 2012 as highlighted in Chart 7.
III. THE ROLE OF LIQUIDITY IN CRISIS

The role of market liquidity in crisis is mainly twofold: the liquidity of an asset grows with bubbles until a critical state where it may suddenly fall in illiquidity and precipitate a crash.

First, speculative bubbles often arise in a frame of a huge turnover of transactions. Cochrane (2003) observes indeed that prices higher than the fundamental value of an asset are associated with high trading volumes and low supply: therefore, the market seems very liquid for buyers, who do not face much difficulty to buy the asset at an increasing price. Baker and Stein (2004) also explain that phenomenon by a model in which a big liquidity is generated by the big number of irrational traders who overestimate the value of the asset. In short, bubbles and liquidity grow concomitantly.

When bubbles have grown a lot, they may burst and the role of liquidity may be determinant. We underlined that when the bubble arises, the volume is high but the supply is low. This means that the situation is quite unstable and can reverse dramatically. The mechanism of crisis involving the market liquidity is also linked with the funding liquidity\(^8\) of the investors. That is the downward liquidity spiral,

\(^8\) For Pedersen (2008), the funding liquidity risk is the risk that the investor is unable to fund his position on an asset from its own capital and is forced to unwind. For example, leveraged investments of hedge funds are possible if banks lend them money, but if banks raise their margins the funding costs increase a lot to such investors and they must deleverage their positions.
as detailed by Pedersen (2008) or Brunnermeier (2009): when the market liquidity begins to be reduced, then prices become lower and the risk management, fearing the upcoming crisis, tightens. Therefore, the funding is more complicated and this implies less transactions. The mechanism loops here, as the diminution of the number of transactions reduces again the market liquidity and worsens the crisis.

Beside these descriptive analyses, an increasing number of papers propose methods to predict the time at which the bubble will burst, like Kaizoji et al. (2002). Most of them describe the behaviour of traders and rely on the assumption that some traders know the fundamental value of the traded asset. This allows these articles to link the price to the supply and demand (which is a consequence of the behavioural mechanism invoked) through the hypothetical knowledge of that fundamental price. Tóth et al. (2011) allow to get rid of such an assumption by taking into account the market liquidity and more precisely the resilience. They describe indeed the price impact of any volume traded. Such a model coupled with this type of bubble model would allow a more realistic description of bubbles and a more accurate prediction of crashes.

In the description of the role of the liquidity in a bubble, we mainly dealt with the liquidity of an asset. However, such a liquidity risk can be spread to the whole market. For instance, the contagion to other asset classes can be led by the funding risk: when margins of the banks raise or even when a bank goes to bankruptcy because of one asset type, investors have difficulties to fund other kinds of investments as explains Pedersen (2008) about the wide bursting of the housing bubble.

9 For instance, risk indicators, such as the VaR, which are calibrated on recent historical price evolutions, are more pessimistic at the first signs of bear market and imply a more conservative attitude.
IMPACT OF LIQUIDITY ON PRICING, RISK MEASURES AND PORTFOLIO CONSTRUCTION

I. PRICING ILLIQUID SECURITIES: MTMARKET VS MTMODEL

In the previously defined interrupted and illiquid markets\(^\text{10}\), the price of the securities is affected by frictions. Moreover, when the illiquidity is particularly strong, the market price of a security may no longer be available. In both cases, getting a fair price, that is a price that reflects all risks except liquidity, is then a useful and challenging purpose.

On the one hand, when a market price is available but is affected by illiquidity, some methods allow estimating the fair price. Among them, Guégan and Merhy (2010), propose to filter optimally, with a Kalman filter, the observed price in order to infer the fundamental price and the related liquidity premium, which is the difference between the observed price and the fundamental one. Their definition of liquidity is similar to that of Chacko and Stafford (2004), who define liquidity as the gap between the fundamental value of a security and the price at which the security is actually transacted; high liquidity means this gap is small and vice versa.

In Guégan and Merhy (2010) method, we must define a fairly general dynamic for the fundamental price. In the aforementioned paper, the fundamental price at time \(t\) is defined as a noisy weighted average of the fundamental price at time \(t - 1\) and the long run price (mainly the par value for fixed income securities). With such a fundamental price model, many situations are taken into account, such as a very erratic random fair price, or a random walk, or a mean-reverting fair price. Chart 9 and Chart 10 illustrate the functioning of their method.

\(^{10}\) Refer to Chart 2.
Liquidity risk in Fixed Income Markets

Chart 11: Liquidity premium dynamics

Liquidity premium switches regime in July 2007 and continues to increase till July 2009 where it begins to ease
Source: Guégan and Merhy (2010)

On the other hand, when illiquidity is such that no market price is available, the only possible thing to do is to refer to liquid markets and find a shrewd proxy. Of course, in that case, as no market price is observable, trying to define a liquidity premium is meaningless. We highlight here two methods which both rely on the same idea that it is necessary to refer to an observable liquid universe of securities. In the following development, we are particularly interested in bonds.

In the first method, we build buckets of bonds reputed to be liquid. Each bucket stands for a particular currency, a sector, a seniority and a credit rating. Then, using a classical bootstrap and a rate curve model such as Svensson model, we build a generic rate curve for such a liquid bucket. Then, when we want to price a particular illiquid bond, we discount its cashflows with the curve of the bucket with the same currency, sector, seniority and credit rating.

Chart 12: Calibrating a model yield curve

Svensson model (in blue) calculated on a bucket of liquid defensive senior bonds of rating A-, as of 5 December 2013. The bootstrapped curve is in pink. The illiquid bonds are then priced with that blue curve
Source: Natixis Asset Management

Generic Yield Curve: EUR Credit NonFin NonFinDef Senior >=A- <=A-

Empirical rate — Rate

16
In the second method, we also build buckets of liquid bonds, but, instead of modeling a generic rate curve, we determine the risk aversion on such a bucket, using the theory of indifference price, following Arrow (1965) and Pratt (1964). Then, similarly to the previous method, we can discount the cashflows of any particular illiquid bond taking into account the risk aversion calibrated on the corresponding bucket.

II. CONTROLLING LIQUIDITY RISK: L-VAR VS VAR

From a risk perspective, ignoring liquidity risk tends to underestimate the overall risk of a position. Bangia et al. (1999) found that ignoring the liquidity effect leads to underestimating of market risk in emerging markets by more than 25%. More recently, Stange and Kaserer (2008) proposed a weighted spread to improve on previous liquidity measures by taking into account the price impact costs into their VaR framework. They found that liquidity factor increases the 10 days VaR@99% by 25% for liquid DAX stocks.

Ernst et al. (2008) distinguish 3 types of models including market liquidity:

- Models based on bid-ask spread data: liquidity costs are captured from observable bid-ask spreads and subtracted from prices. Bangia et al. (1999) and Ernst et al. (2008) developed a liquidity augmented VaR model. This class of models offers the advantage of simplicity. However, only exogenous liquidity costs are taken into account as the price impact is explicitly not modeled.

- Models based on volume or transaction data: this class of models attempt to correct Bangia et al. (1999) drawbacks by estimating the price impact function. For instance, Berkowitz (2000) estimated it from past trades in regression wise approach.


Bangia et al. (1999) included liquidity into a parametric VaR framework. Based on observed bid-ask spreads time series they derived their liquidity augmented VaR model.

Denote by \( r_{t+1} = \ln P_{\text{mid},t+1} - \ln P_{\text{mid},t} \) the logarithmic return of the mid price at time over one period of time. Assuming centered Gaussian returns, Bangia et al. (1999) exploit information embedded in the distribution of normalized spreads to incorporate the effect of liquidity risk into a parametric VaR framework. Prices dynamic is given by the following equation:
where $S_t$ denotes the normalized price spread at time $t$, namely:

$$S_t = \frac{P_{ask,t} - P_{bid,t}}{P_{mid,t}}.$$ 

Given these assumptions, Bangia et al. (1999) derived a close form expression for the liquidity adjusted VaR\(^{11}\). Though their approach relies on restrictive assumptions, its main advantage relies in the low data set required to include liquidity into a risk framework.

III. PORTFOLIO CONSTRUCTION AND STRESS TESTING: THE LIQUIDITY EFFICIENCY SCORE

The \textit{ex ante} future distribution of P&L of the portfolio is the relevant objective function that a portfolio manager looks at and insure that it has the adequate properties especially from a risk perspective. Tracking Error Volatility (TEV) or Value at Risk (VaR) are synthetic risk statistics commonly used to measure the risk of yet to come P&L distribution.

Even if the investor is absolutely certain about the outperformance of a specific strategy in the near future, would he go long this strategy if transaction costs are prohibitive? The answer requires a general framework that explicitly takes into account liquidity risk for portfolio construction.

To account for liquidity risk, conventional VaRs are computed over longer horizon in an ad hoc fashion. The Basel Committee on Banking Supervision (2009) extended from 10 days to 3 months the liquidity time horizon in the calculation of VaR\(^{99\%}\). The Basel Committee on Banking Supervision (2009) acknowledged that the liquidity of traded assets varies substantially over time and that banks’ exposures to market risk and credit risk vary with liquidity conditions in the market.

Bangia et al. (1999) addressed the problem of computing liquidity VaR for a portfolio. They recommend to compute the average portfolio spread and to apply their abovementioned technique in order to avoid computing correlation among bid-ask spreads between securities\(^{12}\).

Meucci (2012) presented a framework for modeling jointly market risk and liquidity risk. Liquidity is not just a deterministic bid-ask but modeled as a risk factor per se whose impact on the P&L of the portfolio is state dependent. Thus when volatility is high and market is down the negative impact of liquidity is more important. The framework takes also into account endogenous liquidity risk stemming from forced selling in adverse market scenarios. The impact of liquidity risk on the portfolio P&L will depend also on the liquidation scheme or the turnover, in a take profit case or a stop loss one.

\(^{11}\) Reader may refer to appendix for a full presentation of Bangia et al. (1999) approach.

\(^{12}\) More recently Brigo and Nordio (2010) accounted for liquidity by introducing randomness into the holding period. The operational time over which assets should be liquidated may differ from the one retained for computing risk measures for instance VaR. More generally, a portfolio manager who rebalances his portfolio on regular basis e.g. monthly may not be able to fully adjust his portfolio to his new set of views if liquidity conditions are degraded and a longer horizon is required. Their stochastic holding period (SHP) shifts the P&L distribution downward and increases VaR.
The choice of a liquidity model in Fixed Income markets will ultimately depend on available data e.g. LCS in our case. Though LCS does not account directly for price impact for large trades, Dastidar and Phelps (2009) argued that it is highly correlated with price impact. It is found to be persistent on average: bonds with low LCS are likely to remain liquid according to the LCS measure for a while.

Our approach differs from that of Bangia et al. (1999) in 3 ways: it departs from the normality assumption, it models the liquidity for each security and aggregate positions holdings into the portfolio and it takes into account dependency structure between traditional risk factors and liquidity ones. Moreover, our non-parametric approach allows for stress testing and scenario analysis.

For each security, we define the achieved return in $h$ steps forward, as a function of the carry, the market return and the liquidity cost. Carry is proportional to the mid YTM and the market return is proxied by the product of the modified duration by the sum of $h$ variations of the mid YTM. Liquidity at the liquidation horizon is assumed as in Bangia et al. (1999) equal to half of LCS$^{13}$ prevailing at the end of the period e.g. $t + h$.

$$L\text{ Ret}_{t \rightarrow t+h} = \frac{YTM_{mid, t} \times \text{Coverage}_{t \rightarrow t+h} - \text{ModDur}_{mid, t} \times \Delta YTM_{t \rightarrow t+h} - \frac{1}{2} (LCS_t + \Delta LCS_{t \rightarrow t+h})}{\text{Projected Return over } h \text{ Periods}}$$

At the beginning of the period e.g. at time $t$, the carry and the coverage are known. We need to project YTM and LCS to the end of the period. We do that by jointly modeling the variations of YTM and those of LCS. For the sake of simplicity, we consider a portfolio of equally weighted generic securities corresponding to the 14 credit sectors$^{14}$ presented in section I-C. The joint distribution of YTM and LCS weekly$^{15}$ changes is calibrated over the period June 2010 till June 2013.

Chart 14: The LVaR@99% over different horizons

The red sticks correspond to the difference between the LVaR and the traditional VaR (in blue)

Source: Natixis Asset Management

Omitting liquidity, underestimates VaR@99% by 22% over the short run. The underestimation decreases with the projection horizon as market risk would take over liquidity as depicted in Chart 14. At a 6 month horizon, the ratio of LVaR@99% to traditional VaR@99% would converge around 10% suggesting that usual market risk factors will take over liquidity risk factor over longer horizons.

13 In Bangia et al (1998) bid-ask spread is normalized to the mid price, LCS is normalized to the bid price and hence is more conservative.

14 The sector partition is the one used by the credit investment team in NAM’s Fixed Income department.

15 LCS are computed by Barclays according to a monthly frequency. Weekly changes are recovered by interpolation and bootstrapping techniques.

The framework allows for stress testing liquidity risk. It allows us to examine the impact on VaR of higher correlation between YTM and LCS, of larger LCS volatility or both.
In Table 3 we reproduce figures we obtain when increasing\textsuperscript{16} the correlation between LCS and YTM everything else being equal.

<table>
<thead>
<tr>
<th></th>
<th>VaR@99%</th>
<th>LVaR@99%</th>
<th>Stressed LVaR@99%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-1.26</td>
<td>-1.54</td>
<td>-1.80</td>
</tr>
</tbody>
</table>

Portfolios can be ordered according to their liquidity profile. For instance, portfolios with lower LCS are preferred over less liquid ones. Meucci (2012) proposed the liquidity efficiency score as a criterion for assessing portfolio’s liquidity risk. Since the liquidity adjustment always hits the P&L downward, he defined the liquidity efficiency score as the percentage of deterioration of the left tail e.g. as the ratio of traditional expected shortfall\textsuperscript{17} to the liquidity augmented one. This ratio is larger than 0 and lesser than 1. The closer to 1, the lower the risk of liquidity. The four-week efficiency score of our sample portfolio is equal to 91%.

**CONCLUSION**

Portfolio managers need to take into account the effect of liquidity on pricing, the cost of transactions etc in the decisions they make. When modelling the liquidity premium, we obtain different results in terms of VaR or expected returns. This implies that very often the concrete decisions portfolio managers make are different from theoretical optimal decisions, precisely because of the lack of liquidity. One of the difficulties is that the lack of liquidity often increases when financial risk or risk aversion increase. This is not a diversifiable risk or a risk that it is possible to hedge. On the other hand, the absence of liquidity can also offer a surplus of return for long-term investors who do not need liquidity from day to day.

\textsuperscript{16} The stressed correlation matrix is a linear combination of the original one and a panic matrix with cells 1 for correlation between LCS and YTM.

\textsuperscript{17} Expected shortfall (ES\textsubscript{99}) is defined as the average loss exceeding VaR\textsubscript{99}. 
APPENDIX

Bangia et al. (1999) included liquidity into a parametric VaR framework. Based on observed bid-ask spreads time series they derived their liquidity augmented VaR model.

Denote by \( r_{t+1} = \ln P_{mid,t+1} - \ln P_{mid,t} \) the logarithmic return of the mid price at time \( t \) over one period of time e.g. from \( t \) till \( t+1 \). Assuming centered Gaussian returns, Bangia et al. (1999) exploit information embedded in the distribution of normalized spreads to incorporate the effect of liquidity risk into a parametric VaR framework. Price dynamics are given by the following equation:

\[
\begin{align*}
P_{mid,t+1} &= P_{mid,t} e^{r_{t+1} - \frac{1}{2} P_{mid,t} S_{t+1}}, \\
\end{align*}
\]

where \( S_t \) denotes the normalized price spread at time \( t \), namely:

\[
S_t = \frac{P_{ask,t} - P_{bid,t}}{P_{mid,t}}.
\]

Under perfect correlation between liquidity and return they derived the following formula for the liquidity adjusted VaR:

\[
\text{LiquidityAdjustedVaR}_{99\%} = P_{mid,t} \left( 1 - e^{-2.33.\theta.\sigma} \right) + \frac{1}{2} P_{mid,t} (\mu_s + K.\kappa),
\]

where \( \sigma \) denotes the volatility of returns and \( \theta \) is a scaling factor. \( \theta = 1 \) for Gaussian distribution and \( \theta > 1 \) to account for fat tailed returns. \( \mu_s \) and \( \sigma_s \) are the mean and the standard deviation of the bid-ask spreads and \( \kappa \) the 99% empirical percentile. They found that it ranges between 2 and 4.5 as compared to 2.33 for the 99% percentile of the Gaussian distribution.

The main advantage of this approach relies in the low data set required to include liquidity into a risk framework. The historical spread series are sufficient. The drawdowns are the additive nature of liquidity risk irrespective to correlation issues especially in the tail dependence. It also fails to take into account the price impact function which leads to underestimation of higher positions.

Ernst et al. (2008) departure from the Bangia et al. (1999) Gaussian assumption for prices and use a Cornish-Fisher approximation to develop their liquidity adjusted total risk VaR. Although their specification yields a more precise risk forecast, it fails to capture the price impact as the liquidation occurs on the bid-ask spread cost and correlation among risk factors is assumed perfect. The advantage is still the same with the additive add on scheme.
REFERENCES


Liquidity risk in Fixed Income Markets

ADDITIONAL NOTES

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