Investing in global convertible bonds – stylized facts, pricing and strategies

*Research paper* #8

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A convertible bond gives its holder the right to convert a bond, which is exposed to credit risk, into a predetermined amount of shares. This attribute makes the convertible bond a hybrid asset between credit and equity. It would thus be tempting to consider it as a compromise between both asset classes. Reality is sometimes slightly different. Indeed, a study of long-memory features indicates that the forecastability of convertible bonds is almost always higher than for stocks, often higher than for investment grade bonds and lower than for high yield bonds. Convertible bonds can in addition diversify an investment grade credit portfolio, depending on the investment horizon, better than high yield bonds do. The return can be further increased by including convertible bonds in a multi-jurisdictional portfolio. This is because there are large dissimilarities among convertible bonds across regions. There are indeed fundamental differences in how convertible bonds are considered in different regions: in the US, they are considered to be equity-like, whereas they function more like debts in Europe. This difference of design across regions leads to a wide variety of exposures to equity.

The valuation of a hybrid security, such as a convertible bond, may be delicate since it mixes different sources of risk: equity and credit. Linking default risk to equity value is not new, but its implementation as a reduced-form model in pricing engines of convertible bonds appeared only roughly a decade ago. An uninterrupted increase in the complexity of convertible bonds with many additional options makes the pricing of convertible bonds more and more challenging. This is even more striking for global convertible funds because specific additional options are embedded in the convertible bonds of each region.

Besides balancing equity and credit views by adjusting the delta, convertible bonds can create performance when considering them as an asset class in their own right. We investigate the long-term performance factors of convertible bonds. We also present some standard strategies based on convertible bonds, such as the hybrid strategy, which consists in investing in their convexity. These strategies are well known by arbitrageurs of convertible bonds. We thus confront the risk-adjusted performance of the hybrid strategy to the investment horizon to depict how long-term investment can rely on such strategies.

In this asset manager’s tour of convertible bonds, we first compare convertible bonds with equity and credit, focusing on risk and diversification. We then consider the differences between convertible bonds of different regions, valuation and finally the diversity of sources of performance.
RISK AND DIVERSIFICATION
PERSPECTIVES OF CONVERTIBLE BONDS

A. COMPARISON WITH EQUITY AND CREDIT

Convertible bonds are often seen as a compromise between equity and credit. By adjusting the delta \( \delta \) of a portfolio of convertible bonds, the portfolio manager tries to balance, on the one hand, the opportunities of growth provided by equities and the secure floor offered by bonds and, on the other, the risk of equity losses and the risk of default. The choice of the delta is related to the forecast of business cycles. When equity prices are going up, as was the case in 2013, the delta should be high so as to take advantage of this increase. On the contrary, when equities are declining, as was the case in 2008, the delta should be lower to ensure a protection by the credit. In order to compare Euro credit, equity and convertible indexes, we can first focus on standard metrics depicting the performance and the risk, reported in Table 1.1.

<table>
<thead>
<tr>
<th></th>
<th>Annual total return</th>
<th>Annualized volatility</th>
<th>Annual Sharpe ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euro IG</td>
<td>3.85%</td>
<td>2.73%</td>
<td>1.41</td>
</tr>
<tr>
<td>Euro HY</td>
<td>7.38%</td>
<td>5.65%</td>
<td>1.31</td>
</tr>
<tr>
<td>Euro convertible bonds</td>
<td>3.83%</td>
<td>6.85%</td>
<td>0.56</td>
</tr>
<tr>
<td>Euro equities</td>
<td>5.35%</td>
<td>20.05%</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The Euro convertible bonds and the IG credit index have close historical performances. However, convertible bonds have a higher volatility, which is consistent with the fact that a portion of the issuers of convertible bonds are rated HY. Incidentally, the performances of Euro HY bonds and equities are higher than the performances of Euro convertible bonds and IG credit. The Sharpe ratio, which is a risk-adjusted performance, allows a fairer comparison than the sole total return. Indeed, despite the higher performance of the Euro equity index over convertibles during the time interval studied, the Sharpe ratio takes into account the very high risk associated to it and leads to a value of 0.27, which is the lowest of the four indexes. According to this analysis, convertible bonds are a compromise between credit and equity, since their Sharpe ratio is higher than the equity ratio and lower than the ratios for both HY and IG bonds.

Such metrics are a snapshot at a given moment and they reduce a complex problem to a single number. We have thus attempted to obtain a more solid view on the subject using more sophisticated statistical properties of the time series of convertible bond prices. In particular, we now focus on two metrics. The first describes the statistical memory in the series of price returns and thus the ability of the portfolio manager to make reliable predictions. The second analyzes the ability of convertible bonds to diversify a credit or equity portfolio.

1. Memory in the series of price returns

The performance of a portfolio invested in equities, bonds or convertible bonds is determined by the ability of the portfolio manager to make good forecasts. However, the evolution of the price of financial assets is very uncertain, so that...
the allocation and the performance of the portfolio are based on two distinct parts: one part that can be forecasted and the other that is a factor of chance. The first one always leads to good performances if the decisions of the portfolio manager are consistent with its forecast, whereas chance implies either gains or losses. Equities are reputed to have a narrower forecastable part than bonds have. The question is: how large is the forecastable part of convertible bonds? Is it between equity and credit?

The Hurst exponent is an appropriate statistic to answer this question. Details on the Hurst exponent are available in the appendix of this paper. Intuitively, $H$, the Hurst exponent of a time series, measures the long-term memory of a time series. Its value lies within a range of 0 to 1. The higher $H$ is, the more prominent is the long-memory effect. If $H = 1/2$, there is no memory: two consecutive returns are independent and forecasts are hence impossible. If $H < 1/2$, increments are anti-persistent. It means that the next return is negatively correlated with previous returns. Forecasts are certainly possible in this case, but one needs to trade at the right time and frequency since gains are quickly erased by anti-persistence. This seems appropriate for high-frequency trading. For an asset manager with a longer investment horizon, anti-persistence is rather opposed to forecastability. Last but not least, increments are persistent if $H > 1/2$, which implies a smooth evolution of returns. Therefore, if $H$ is known, the forecastability of the time series will be increased for higher Hurst exponents.

We show the evolution of the Hurst exponent for Euro equity, credit (IG and HY) and convertible indexes in Figure 1.1. Generally, it is almost always lower for equities than for convertible bonds, and very often lower for equities than for IG bonds. This underlines the difficulty in forecasting equity returns over long time periods, according to this statistic. The superiority between convertible bonds and IG credit in terms of forecastability depends on time.

![Figure 1.1: Dynamic Hurst exponent, estimated on a 300-business-day sliding window](source: Barclays, Exane, Bloomberg, Natixis Asset Management)

The dotted line delimits two zones: long-memory (above) and anti-persistence (below).

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2. Christoffersen and Diebold (2000) show for example that the volatility of a stock index is forecastable up to a horizon of 10 days, whereas the forecastable horizon goes up to 20 days for government bonds. In addition to this example, we will more precisely link the forecastability of a price series (and not only of volatility) to a long-memory metric.

3. Many papers illustrate this intuitive link between long memory and the forecastability of a financial time series. For example, we find such an idea in Geweke and Porter-Hudak (1983). More recently, Qian and Rasheed (2004) empirically showed that the higher $H$ is, the more forecastable prices are. In particular, they analyzed such a phenomenon on various periods of the Dow-Jones index between 1930 and 2004: if the Hurst exponent is high in a given period, the prices are more easily forecastable. Similar profitable trading opportunities linked to high Hurst exponents are also documented by Mitra (2012) and Garcin (2016).
However, the source of uncertainty is threefold:

- The value of $H$ indicates whether forecasts are reliable, as previously explained.
- We calculate a dynamic Hurst exponent, so that the value of the local Hurst exponent which underlies the next return is unknown, even though an assumption of smooth evolution can sometimes be relevant.
- The estimation of $H$ is itself uncertain. We can build approximate confidence intervals as explained in the appendix. These confidence intervals show for example that an estimated Hurst exponent of 50% is in reality in the range [46%,61%] with a 90% confidence level. The last reported value of the Hurst exponent in Figure 1.1 should thus be interpreted carefully: 53.9% for convertibles, 60.2% for IG bonds and 50.2% for stocks. However, the last Hurst exponent of the HY bonds, 70.9%, seems significantly higher than for the other asset classes.

The empirical distribution of the Hurst exponent of each kind of asset makes it possible to gauge the predictability of the next local Hurst exponent. Therefore, a spread distribution creates more uncertainty regarding the second source of uncertainty given above. They are represented in Figure 1.2. It seems that this distribution is less spread for convertible bonds than for IG credit, even though, IG credit can reach larger Hurst exponents (but also smaller ones). Last, we mentioned that forecasts are easier when $H$ is above 1/2. In the empirical distributions, the probability that $H > 1/2$ is equal to 90.3% for convertible bonds versus 80.4% for IG credit. Globally, it thus seems that, from 2006 until 2016, returns are more easily forecastable for Euro convertible bonds than for Euro IG credit. This conclusion is surprising since convertible bonds could be expected to behave like a compromise between equity and credit, not like assets that were more forecastable than both equity and IG credit.

Besides, due to high coupon rates, the Hurst exponent of the HY bonds is almost always higher than IG bonds, stocks and convertible bonds. In the convertible index, convertibles are often not rated and rated convertibles may be either IG or HY. As a consequence, the credit quality of the index of convertible bonds is between IG and HY. It may however be interesting to compare convertible bonds with IG credit only, as a possible source of diversification of IG credit.

On average, convertible bonds and IG credit have the same forecastability. However, the span of reachable Hurst exponents is narrower for convertible bonds. This results in more frequent forecastable returns ($H > 0.5$) than for IG credit.

### Figure 1.2: Estimated probability density of the dynamic Hurst exponent since 2006

*Source: Barclays, Exane, Bloomberg, Natixis Asset Management*

> **2. Convertible bonds as a source of diversification**

We now analyze the correlation of returns between convertible bonds and credit, or between convertible bonds and equity. We calculate it at different scales thanks to the multiresolution correlation, which is detailed in the appendix.
This multiresolution analysis allows us to determine the horizons to which convertible bonds can be considered as acceptable assets in order to diversify a credit or equity portfolio.

We show this multiresolution correlation in Figure 1.3 for the Euro universe. We notice a high correlation, of around 80%, between convertibles and equities whatever the scale. Therefore, using convertible bonds in an equity portfolio does not on average constitute a diversifying strategy. At the other end of the scale, the correlation between convertibles and IG credit is far lower. It globally increases with the scale. For daily returns, the correlation is even negative. It remains below 60% for a horizon of roughly one month or less.

![Figure 1.3: Multiresolution correlation](image)

The multiresolution correlation of convertibles with HY bonds is significantly higher than its correlation with IG bonds. In addition, the multiresolution correlation between HY and IG is also higher than between convertibles and IG. This means that convertible bonds constitute a more efficient source of diversification for a portfolio of IG bonds than HY bonds do, according to our sample. This finding offsets the higher forecastability of HY bonds in comparison to convertible bonds, based on the analysis of Hurst exponents.

**B. AMONG CONVERTIBLE BONDS: THE GEOGRAPHICAL SPLIT**

In the previous section, we set out the extent to which convertible bonds can be considered as a source of diversification for bonds and equity. In addition, the great diversity of profiles among convertible bonds makes it possible to build balanced portfolios of convertible bonds. In this section, we show how this applies to a global universe of convertible bonds. We will quantitatively justify the intuition that a global universe offers more possibilities for a portfolio manager than does the sole European universe. This diversity can be appreciated either by the correlation between performances of the sub-indexes or by comparing certain features of the constituents of the sub-indexes.

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4. The confidence interval assumes a Gaussian distribution of the Fisher transformation.
5. The index considered is the Thomson Reuters Global Focus Index.
1. Dissimilarity of the constituents of regional sub-indexes

The diversity among regional universes can be illustrated by the difference in width of the ranges of deltas. In Figure 1.4, we present the empirical density of deltas in each of the four regions of a global index on 31 March 2016. This indicates how enlarging the universe of convertible bonds makes it possible to reach certain ranges of deltas. For example, it is easier to find a high-delta convertible bond in the US, next in Japan, in the rest of Asia and finally in Europe, whereas bond profiles are more frequent in Europe than in the US. The spatial standard deviation of the European distribution of deltas (17.7%) also increases when considering the global distribution (19.7%). This indicates that there are more opportunities when managing a global portfolio than a European portfolio.

![Figure 1.4: Empirical density of deltas in the global index as of March 31st 2016](source: Thomson Reuters, Natixis Asset Management)

The dissimilarity between the different regions in the global universe is a source of opportunity for implementing a strategy. This dissimilarity between empirical probability densities is measured by the Hellinger distance. This metric goes from 0 (strong similarity) to 1 (strong dissimilarity). For densities $f$ and $g$, the Hellinger distance $H(f, g)$ is defined by:

$$H(f, g) = \sqrt{\frac{1}{2} \int \left( \sqrt{f(x)} - \sqrt{g(x)} \right)^2 dx}.$$  

In our framework, densities are estimated by a Gaussian kernel. This estimate relies on the arbitrary choice of a parameter, the bandwidth. The higher the bandwidth, the smoother the estimated density. However, if the bandwidth is too high, the distribution does not look realistic. On the other hand, if the bandwidth is too low, the estimated density will be overfitted. Reality should be in between.

With the notable exception of some very low deltas in the Asian market, the European market is more bond-like since deltas are lower than in the rest of the world. The US market has the highest deltas.
Therefore, we present the Hellinger distance for a wide range of bandwidths in Figure 1.5. It shows three groups of pairs with different kinds of dissimilarities. To put it simply, the dissimilarity regarding the deltas between Europe and each of the other regions is greater than for any other pair of regions. The most similar regions are the US and Japan. It thus confirms that extending a European universe to a global universe is relevant, since the dissimilarity of the European universe with the rest of the world is particularly great and hence makes new strategies possible.

Figure 1.5: Dissimilarity (Hellinger distance) of distributions of deltas for pairs of regions, for various bandwidths (the bandwidth used in Figure 1.4 is 0.01)

Source: Thomson Reuters, Natixis Asset Management

Dissimilarity of regions regarding their distributions of deltas is promising when constructing the strategy of a portfolio. Such an analysis can also be done for statistics other than the delta. For example, studying the dissimilarity between the distributions of remaining maturities can be relevant. We thus see in Figure 1.6 that the US and the Japan universes have more convertible bonds with long maturities, whereas short maturities can more easily be found in the Europe universe.

Figure 1.6: Empirical density of remaining maturities in the global index as of March 31st 2016

Source: Thomson Reuters, Natixis Asset Management

Densities are estimated by a Gaussian kernel.
The corresponding Hellinger dissimilarity measures, as shown in Figure 1.7, indicate that the US have a strong dissimilarity with other regions, whereas the similarity between Japan and Europe or between Japan and the rest of Asia is quite high.

Dutordoir and Van de Gucht (2004) confirm these results concerning the differences between European and US convertible bonds. They observe that the European convertible bonds issued in the period 1990-2002 are more debt-like than the US convertibles issued in the same period. Beside the fact that US investors generally consider convertible bonds as equity-related whereas European investors consider them as debt-like, Dutordoir and Van de Gucht noted a number of features in the design of convertible bonds highlighting this dissimilarity. Compared to European convertible bonds, in general US convertible bonds have a higher delta, a longer maturity at issuance and are more often callable. These three characteristics increase the probability of conversion of the bond before its maturity.

2. Correlation of the returns of the regional sub-indexes

The dissimilarity between regions in terms of constituents is one way of highlighting the relevance of a global universe. The correlation of the performance of each regional sub-index is another way. We depict this with Spearman’s rank correlation coefficient, known as Spearman’s rho. This statistic measures the correlation between two series of returns without assuming that there is a linear relationship between them as implied by a traditional Pearson’s correlation. If Spearman’s rho is 1, then there exists an increasing function linking both series.\(^6\) If Spearman’s rho is -1, then both series are linked by a decreasing function.

We calculate Spearman’s rho for monthly returns between December 1993 and December 2014 for pairs of the four regional sub-indexes mentioned earlier. All the pairs of series of returns are positively correlated. The correlation is very strong between Europe and the US (69%). This means that adding the US to the European universe will not diversify this universe very much regarding the average performance. The correlation is also quite strong for two other pairs of regions:

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\(^6\) This is true whatever the function, whereas it is limited to linear functions for Pearson’s correlation.
Asia ex Japan vs US (58%) and Asia ex Japan vs Europe (55%). Finally, the three other pairs are more likely sources of diversifications: Japan vs Europe (43%), Japan vs Asia ex Japan (41%) and Japan vs US (38%). Confidence intervals justify the repartition of the pairs of regions in the three groups of correlations mentioned above. The minimal spacing between two of these groups is indeed roughly the radius of a 90%-level confidence interval.\(^7\)

Regarding only the performance, a European portfolio manager will mainly be interested in diversifying his portfolio with Japanese convertible bonds. This complements the discussion in the previous paragraph. Indeed, the dissimilarities between the constituents of the sub-indexes suggest that American convertible bonds are also a very good source of diversification of European convertibles.

II PRICING OF A CONVERTIBLE BOND

In order to implement performing strategies or to make accurate predictions, an exact pricing is a strong prerequisite. For convertible bonds, it is even more challenging that two sources of risk, namely equity and credit, are mixed and that many additional and sometimes complex options can be embedded. We have set out below to give an overview of the specificities of the valuation of a convertible bond and review how one can deal with these features.

A. BASIC FEATURES

A convertible bond is characterized by its conversion ratio. At each point in time where conversion is allowed, the convertible bond may be converted by its holder into \(K\) shares (where the share is predefined: usually, this is the share of the issuer in the standard case but for exchangeable bonds it will be another share), where \(K\) is its conversion ratio. Therefore, at maturity of the bond, if conversion is still allowed, the payoff is similar to the payoff of \(K\) call options on the share. At earlier dates, the pricing of the convertible bond provides a mixture between the actualized payoff of the convertible bond, the probability of a jump to default and additional options which can be exercised during its life. The two first basic features (the basic pricing and the hazard rate) are presented in the paragraphs below. We will mention the more specific feature of the additional options later.

1. Pricing of the convertible bond

Before 2003, the mainstream model for pricing convertible bonds was based on a binomial tree. This method was proposed by Cox, Ross and Rubinstein (1979) for option pricing. It was extended to convertible bonds by Bardhan et al. (1994). The root of the tree corresponds to the current price \(S\) of the underlying share. This node leads to two other possible nodes one time step further: with a probability \(p\), it leads to the upper node, which corresponds to an increase of the price \((uS)\), and, with a probability \(1-p\), it leads to the lower node, associated to a decrease of the price \((dS)\). Such a tree is recombinant since if the price goes up and down it produces the same price as if it goes first down and then up. The triplet \((p,u,d)\) is chosen so as to reproduce the variance \(\sigma^2\) of the share price as well as the risk-free rate \(r\):

\(^7\) The confidence interval assumes a Gaussian distribution of the Fisher transformation.
where \( r \) is the duration of the time step. The share price evolution is thus modelled until the maturity of the convertible bond. Depending on the final value of the share, the payoff of the convertible bond is calculated for all the terminal nodes. The price of the convertible bond is then deducted for every node one time step backward as the expected value of the two associated nodes, actualized by the risk-free rate augmented by a spread if there is no conversion. Besides, we suppose that the bond may be converted at each time and, therefore, the prices in the nodes associated to a given time are replaced by a conversion price if it leads to a higher value than without conversion. Finally, after iterating this procedure for each time, we go back to the root of the tree and obtain the current price of the convertible bond. To sum up, the tree is a probabilistic representation of the future share price at different dates. At each date and price of the share, it is decided whether to convert the bond (in which case the convertible bond directly relates to the simulated share price) or to keep it unconverted (in which case the convertible bond price is the actualized expected price of the convertible bond), depending on which of these decisions is most valuable.

The pricing models evolved when the stock risk was no longer considered to be the sole relevant risk. Indeed, credit risk may also have major effects and, in addition, it can be related to the share price. In the tree-based model, the share price was not exposed to credit risk since a risk-free rate was used for conversions. Whatever the exact nature of the link between credit risk and stock price, a risk of jump to default has to be introduced at each time step. This can be done by overloading the binomial tree but, in this case, one loses the main advantage of the tree: its simplicity. The evolution of computers made it possible to model the stock price diffusion directly as well as the price of the convertible bond by a stochastic differential equation (SDE) and to solve it numerically by finite difference equations. This jump-to-default approach was detailed by Ayache, Forsyth and Vetzal (2003), Andersen and Buffum (2004) and Frank (2009). For each time \( t \), a hazard rate \( h(t) \) is calculated. When a default occurs during a step of duration \( dt \), what happens with a probability \( h(t)dt \) conditionally to the fact that it has not occurred before, the convertible bond price jumps to its recovery value \( D \), which is:

\[
D = \max\{RF, KS(1 - \eta)\},
\]

where:
- \( F \) is the facial value of the bond,
- \( R \) is its recovery rate in case of default,
- \( K \) is the conversion ratio,
- \( S \) is the share price,
- \( \eta \) is the rate of loss for the share if the bond is defaulted.

In other words, in case of a default, the price is either the recovery price of the bond or the conversion price, given that the share price is also affected by the default.

Figure 2.1 presents the price of a convertible bond depending on the price of the underlying share. The bond is a standard convertible bond without any additional options.
It is priced using the SDE approach, with a hazard rate linked to the share price, as explained in the next paragraph. The risk-free rate is assumed to be equal to zero and the recovery rate in case of default is 40%. The same convertible bond, with the same market conditions, is priced 5 years before its maturity (in purple) and 1 year before its maturity (in blue). We observe several regimes corresponding to different stock prices: for very low stock prices, the credit risk is sensitive and induces prices below par; in this case, the convertible bond behaves like a bond with low credit risk and is priced around 100 before growing concomitantly with the stock price so as to have a stock profile for high stock prices. In between, there is a regime where the stock price is close to the conversion price in which the curve has a high convexity. This regime is interesting for strategy purposes as is detailed in the dedicated section of this paper. Besides, the longer the maturity, the smoother the curve of prices. Therefore, the convertible bond with the longer remaining life has a higher price than the bond with a shorter maturity, except when the stock price is low enough to induce a notable credit effect.  

![Figure 2.1: Calculated convertible bond price for different stock prices](source: Natixis Asset Management)

The bond has an annual 2% coupon, paid twice a year. The conversion ratio is 1. The stock has a volatility of 20%. The market conditions are unchanged between the following two cases: 5 years remaining (in purple) and 1 year remaining (in blue).

2. Hazard rate

The hazard rate is an essential part of the pricing of a convertible bond. It can for example be estimated by matching the prices for CDS. However, for a given issuer, the single-name CDS may not exist. In this case, one could replace it with aggregated CDS data, but the specific features of the issuer would not appear.

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8. In this example, in order to have the blue curve below the purple one for all the stock prices, the hazard rate for the blue curve would have to be modified, for example by multiplying it by a value at least equal to 4.2.
Alternatively, the hazard rate can be linked to the share price. The rationale can be related to the Merton model. In his framework, Merton (1974) equates the financial value of a corporation with the sum of the debt value and the equity value. Therefore, if the equity value reaches 0, this means that the financial value of the company would not be enough to redeem the debt. Merton thus estimates the default probability as the probability that the equity reaches 0 before a given horizon, for a Gaussian diffusion of the financial value of the corporation.

The drawback of Merton’s method is that one needs a good knowledge of the financial value of the company and of its dynamic. Practitioners hence prefer to use a simpler approach, which remains consistent with Merton’s rationale. For a given convertible bond, they indeed map the hazard rate, which is the intensity of the jump-to-default process, to the underlying share price. For Takahashi, Kobayashi and Nakagawa (2001), Ayache, Forsyth and Vetzal (2003) or Andersen and Buffum (2004), the mapping is defined by:

\[ h(t) = \frac{b}{S^p}, \]  

where \( b \) and \( p \) are parameters to be estimated. For example, in the case studied in Figure 2.1, we arbitrarily chose \( p=1.3 \) and \( b=1.7 \). Actually, an estimation of these parameters can be achieved as the resolution of an optimization problem minimizing the difference between the market price and the model price. We thus get implicit hazard parameters. This is the method advocated by Zabolotnyuk, Jones and Veld (2010). They estimated those parameters on a sample of 57 actively traded Canadian convertible or exchangeable bonds listed on the Toronto Stock exchange at the beginning of November 2005. The mean, median, minimum and maximum parameters they estimated were: for \( p \), 0.32, 0.18, -4.81 and 5.91; for \( b \), 0.13, 0.07, 0.00 and 0.88. This gives an idea of what values are realistic for these parameters. It would be tricky to interpret the value of \( b \) since it strongly depends on the conversion price: similar convertible bonds with the same hazard rate but with a different conversion price would have the same \( p \) but a different \( b \).

The parameter \( p \) depicts the speed at which the hazard rate varies when the share price changes. When \( p \) is negative, the hazard rate is higher for greater share prices. This is counter-intuitive and bounds should therefore be imposed on \( p \) so as to have comprehensible implicit hazard parameters. The case \( p=0 \) is also inconsistent with Merton’s intuition since the default probability then does not depend on the share price. As a consequence, we should have \( p>0 \).

The hazard rate can also be refined by inserting a stochastic process in addition to the dependence on the underlying share price, as proposed by Kovalov and Linetsky (2008). This increases the computational complexity of the valuation by increasing the dimension of the diffusion, since the share price or the interest rate can also be considered as stochastic.

**B. SPECIAL FEATURES**

The basic feature of convertible bonds takes into account two specific potential events: a conversion and a default. However, since convertible bonds exist, many options have been added to them. These options are now quite standard and make convertible bonds more complex. Their impact is in general not negligible and we can therefore not exclude them when valuing convertible bonds.

1. **Additional options**

The sole conversion feature of a convertible bond is quite straightforward in pricing models. Under some simplistic assumptions, Ingersoll (1977) even proposed a closed formula. However, options are usually added to the convertible bonds,
Investing in global convertible bonds – stylized facts, pricing and strategies

sometimes making the pricing tricky. They may be standard, such as a call or a put, or path-dependent, such as a soft call. Soft calls make calculations complex and the recourse to an approximation may be useful.

A basic option is the 20-of-30 soft call. This means that the convertible bond may be called by the issuer if the underlying share price is above a given trigger at least 20 days out of 30 consecutive days. This is a complex option which is strongly path-dependent. Therefore, the Monte Carlo method is slow when integrating it. The exact solution for a $M$-of-$N$ soft call by partial differential equations needs a time proportional to $2^N$. For the basic case 20-of-30, this means that the calculation is roughly one billion times longer than without such an option. As a consequence, an exact solution is in general not suitable and approximations are necessary. Among other methods presented by Firouzi and Ayache (2015), approximating this option by an $M$-of-$M$ soft call, which is exercised for $M$ consecutive days above the given threshold, leads to a linear computational complexity, which is by far faster than the exact solution.

The particular case of the soft call option illustrates the complexity of pricing convertible bonds.

2. Geographical particularities

We already mentioned in Section I.B.1 that US convertible bonds were considered as equity-like assets, both by the banks which design them and by the investors, whereas European convertible bonds are more likely to be considered as debt-like assets. As a consequence, US convertible bonds are more often callable than European ones, according to Dutordoir and Van de Gucht (2004). However, some geographical particularities can be more striking. Global investors should be aware of these particularities when optimizing their portfolios. We list some of them in the following paragraphs and summarize them in Table 2.1, in which we counted the occurrence of the particularities by region in the Thomson Reuters Global Focus Index on 31 March 2016.

Table 2.1: Proportion of convertible bonds with the specified peculiarity for each region

<table>
<thead>
<tr>
<th></th>
<th>America</th>
<th>Asia ex Japan</th>
<th>Europe</th>
<th>Japan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Valuation in units</td>
<td>-</td>
<td>-</td>
<td>20%</td>
<td>-</td>
</tr>
<tr>
<td>Call option</td>
<td>13%</td>
<td>-</td>
<td>-</td>
<td>7%</td>
</tr>
<tr>
<td>Soft call option</td>
<td>20%</td>
<td>89%</td>
<td>80%</td>
<td>26%</td>
</tr>
<tr>
<td>Put option</td>
<td>36%</td>
<td>84%</td>
<td>18%</td>
<td>14%</td>
</tr>
<tr>
<td>Anytime put option</td>
<td>7%</td>
<td>11%</td>
<td>2%</td>
<td>7%</td>
</tr>
<tr>
<td>Reset option</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5%</td>
</tr>
</tbody>
</table>

a) Valuation in percentage of the notional or in units

A striking geographical difference between convertible bonds exists outside the rich field of specific options. It consists in different valuation conventions. Some convertible bonds, mainly those issued in France, are priced in units, whereas other convertible bonds are priced as a percentage of the notional. One has to consider this carefully when testing a strategy, for example. Indeed, we show in Section III.A.1 how such a difference leads to slightly different ways of measuring the performance. However, this is only a matter of scale. The following paragraphs will present specific options which can lead to unusual responses to movements of the underlying.
b) Soft call options

The usual soft call option gives the issuer the right to redeem or force the conversion of the convertible bonds if their price is above a given threshold during 20 out of 30 consecutive days. However, some soft calls differ from this widely used definition. For example, Lau and Kwok (2004) reported that Chinese soft calls tend to feature a 20-out-of-20 (or 30-out-of-30) condition, with the additional condition that the last of the 20 (or 30) prices lies within a particular range depending on the preceding 20 (or 30) days: for example not less than the fifth lowest price of the last 20 days and not more than all the prices of the last 20 days.

In our sample, we observe that European and Asian (excluding Japanese) convertible bonds often feature a soft call (more than 80% of them). On the contrary, less than 30% of American and Japanese convertible bonds include such an option. However, in these regions, a traditional call option sometimes appears. This is not the case elsewhere. Concerning the design of the soft calls, there is a wide variety, which is reported in Table 2.2. If we focus on the two main regions which provide soft calls, Europe and Asia (excluding Japan), our findings confirm the fact pointed out by Lau and Kwok (2004): the diversity is great in Asia, with almost as many $N$-out-of-$N$ soft calls as $M$-out-of-$N$ soft calls, for $M < N$. On the contrary, the standard in Europe is $M$ out of $N$.

| Table 2.2: Proportion of convertible bonds among soft-call convertibles for each region |
| Source: Thomson Reuters, Bloomberg, Natixis Asset Management |
| **Region** | America | Asia ex Japan | Europe | Japan |
| 15 out of 15 | - | 6% | - | - |
| 20 out of 20 | 33% | 29% | 2% | 73% |
| 30 out of 30 | - | 12% | - | 9% |
| **Total “N out of N”** | 33% | 47% | 2% | 82% |
| 10 out of 20 | - | - | 2% | - |
| 20 out of 30 | 67% | 53% | 79% | 18% |
| 20 out of 40 | - | - | 17% | - |
| **Total “M out of N”** | 67% | 53% | 98% | 18% |

It is worth noting that embedding a call option in a convertible bond decreases its value. Indeed, such an option protects the issuer from a rise in the share price.

c) Put options

An additional put option for the holder is common in Asia excluding Japan and not so much in the other regions. If the holder decides to exercise the option, the redemption can be made either in cash (hard put) or partly in shares (soft put), at the issuer’s discretion. The option can be exercised at predetermined discrete times, potentially on a single date, or anytime.

Embedding put options in a convertible bond protects the holder if the share price declines. It even transforms the nature of the bond floor, which, in absence of such a put option, is affected by the default risk. When, in pricing models, the hazard rate is linked to the share price, as suggested in Section II.A.2, this reflects the probability of a jump to default before the maturity of the convertible bond, given the equity value of the issuer.
But if the issuer progressively goes to default, then the equity value progressively declines to zero and the share price crosses the exercise value of the put option, protecting the holder from the default. In compensation, this protection leads to higher prices for the convertible bond.

d) Reset options

Like put options, reset options offer protection to the bond holder and thus result in higher prices. The reset clause gives the holder the right to reset the conversion ratio upward, which is equivalent to converting the bond into shares for a lower price. The philosophy of this option is close to that of a soft put. In Table 2.1, this option appears to be specific to Japan. Kimura and Shinohara (2006) showed that the impact of such an option on the price of the convertible bond is substantial. They also found that there exists an optimal reset time, which lies within the second half of the life of the convertible bond.

III CREATING PERFORMANCE WITH CONVERTIBLE BONDS

In the previous sections, we analyzed the extent to which convertible bonds mix equity and credit in terms of risk and diversification as well as of optional features and pricing. We showed that, instead of considering convertible bonds as a compromise between a corporate bond and a share, they should rather be seen as a constituent of an asset class in their own right. This is striking when considering that the long-memory characteristics of convertible bonds go beyond a simple average of equity and credit, or when considering the numerous embedded options, which drive the convertible bond away from the sole conversion feature. As a consequence, the creation of performance is specific to this asset class. We hence focus first on certain standard strategies and then on long-term performance factors.

A. CERTAIN STANDARD STRATEGIES

We review some standard strategies using convertible bonds: the hybrid strategy and volatility arbitrage. We more particularly illustrate the relevance of the hybrid strategy, depending on a threshold of Gamma and of the investment horizon.

1. Hybrid strategy

a) Principle

Section II above emphasized that the payoff of a convertible bond is such that, when the underlying share price increases, the convertible bond progressively moves from a bond profile to a stock profile. This evolution is well depicted by the Gamma of the convertible bond. Indeed, a high Gamma indicates that when the share price goes up, this contributes to higher gains than the losses for the convertible bond resulting from a decline of the share price of similar amplitude. The highest Gamma therefore corresponds to a good balance between a stock-like gain opportunity and the protection provided by a bond floor.

However, a lower Gamma corresponding to a higher delta generally leads to much higher gains if the underlying share price goes up. The performance of a high-

9. As a consequence, the absence of put option slightly steepens the bond floor, hence increasing the delta of the convertible bond for similar values of the underlying. Thus, Asian convertible bonds, which massively embed a put protection, can more easily reach lower deltas than other convertible bonds. This confirms the findings of Section I.B.1 concerning the dissimilarity of the distributions of deltas among the four regions.
Gamma convertible bond is therefore not necessarily the most attractive among a number of convertible bonds. The hybrid strategy avoids this delta effect by combining a long position in one high-Gamma convertible bond and a short position in the linear exposure of this convertible bond to its underlying share. Such an exposure is obtained thanks to the delta and the conversion ratio.

b) Performance measure

In the most general terms, the return of a strategy is the ratio of the cash generated versus the cash invested. For long-short strategies, such as the hybrid strategy, such a ratio may not be defined or may be absurd when the values of the long and short positions are close. However, the P&L alone is not an interpretable metric of the performance of the strategy as it is not scaled. In order to address this problem, the usual solution is to calculate a ratio, in which the numerator is the P&L and the denominator the initial value of the convertible bond. This return will have different forms, depending on the pricing convention of the convertible bond:

⇒ If the bond is priced as a percentage of the notional, then the return of the strategy between \( t \) and \( t+T \) is:

\[
\frac{P_{t+T} - P_t}{P_t} \cdot \frac{100 \delta (S_{t+T} - S_t)}{P_tC}\]

where \( P \) and \( S \) are respectively the price of the convertible bond and of the share, \( C \) is the conversion price \( ^{10} \), and \( \delta \) is the delta of the convertible bond.

⇒ If the bond is priced in units, which is the case for French convertible bonds, the return is defined by the following formula, in which \( K \) is the conversion ratio:

\[
\frac{P_{t+T} - P_t}{P_t} \cdot \frac{\delta K (S_{t+T} - S_t)}{P_t}.
\]

c) Risk-adjusted performance measure

In order to test the relevance of such a strategy, we not only have to compare the average return of high-Gamma convertible bonds versus lower-Gamma convertible bonds. Indeed, depending on the average return of the stocks, lower-Gamma bonds, such as high-delta bonds, can produce higher returns. However, high-Gamma bonds are supposed to offer a better protection against stock drawdowns. Therefore, the average return or symmetric risk-adjusted performance measures alone, such as the Sharpe ratio, are not good metrics in our framework, as they do not reflect the asymmetric nature of the strategy. We will hence use three other metrics, detailed in the following paragraphs.

The upside-potential ratio, introduced by Sortino, van der Meer and Plantinga (1999), aims to measure upside performance relative to downside risk. It is defined by:

\[
U = \frac{E\{\max(r - r^*, 0)\}}{\sqrt{E\{\min(r - r^*, 0)^2\}}},
\]

\(^{10}\) The conversion price is the ratio of the nominal versus the conversion ratio.
Investing in global convertible bonds – stylized facts, pricing and strategies

where $E$ is the expectance, estimated historically on the sample, $r$ is the return and $r^*$ the minimal acceptable return, taken here to be equal to 0. It is a refinement of the Sortino ratio, which is itself a refinement of the standard Sharpe ratio. The Sortino ratio is defined as follows:

$$S = \frac{E\{r - r^*\}}{\sqrt{E\{\min(r - r^*, 0)^2\}}}.$$  \hspace{1cm} \text{eq. 8}

The Sortino ratio takes into account the global performance, whereas the upside-potential ratio only focuses on positive returns. The asymmetry between gains and losses can also be measured by a metric based on the partition between gains and losses, weighted by the cumulative distribution function of the returns, $F$. It is known as the Omega ratio and was introduced by Keating and Shadwick (2002):

$$\Omega(r^*) = \frac{\int_{r^*}^{\infty} (1 - F(r))dr}{\int_{r^*}^{\infty} F(r)dr}. \hspace{1cm} \text{eq. 9}$$

For $r^*$ equal to zero and a probability density function with a compact support, two integrations by parts transform the Omega ratio into the ratio of the expected gain versus the expected loss:

$$\Omega(0) = \frac{E\{\max(r, 0)\}}{E\{-\min(r, 0)\}}. \hspace{1cm} \text{eq. 10}$$

Both the Sortino ratio and the Omega ratio are particular cases of the Kappa ratio (see van Harlow (1991)), whose objective is to generalize the usual measures of a risk-adjusted performance taking into account the skewness of returns.

In the rest of this paper, we will compare $U$, $S$ and $\Omega(0)$ for different quantiles of Gamma. The higher the three ratios, the more efficient the strategy in terms of gain adjusted from the losses.

d) Backtest

Below is set out the way the strategy was tested to see if it was really relevant and how it should be used.

The strategy was tested on a sample of convertible bonds formed by the components of the Exane Euro index. We noted the price and the Gamma every day between August 2013 and March 2016. In this period, there were on average 36 convertible bonds in the index. Using the set of all the calculated Gammas, we built an empirical distribution of the Gammas and observed the performance ratio corresponding to different classes of Gamma such as quantiles of Gamma.

First, it was ascertained that the asymmetric performance ratios presented above were actually high for higher Gammas. As shown in Figure 3.1, the three ratios did indeed increase for the hybrid strategy for high Gammas and are particularly high for approximately the 10% highest Gammas. This proves the relevance of the rationale behind the hybrid strategy. In addition, the three ratios are very low for the 5% lowest Gammas. It corresponds to credit risk: the risk dominates the stock-related opportunities.

11. In the present framework, this cumulative distribution function is estimated by the discrete empirical one.
12. Note that only $S$ can be negative.
13. A ratio is high in comparison to what is observed for other groups of Gamma. In addition, there are some absolute thresholds defined by the rationale. In particular, an Omega-0 ratio of 100% indicates an average gain equal to an average loss. Therefore, a value above 100%, as observed for high Gammas, is the sign of a positive asymmetry of the performance.
Secondly, we tested a similar strategy corresponding to the sole long part of the hybrid strategy, that is to say without getting rid of the delta part by selling an adjusted amount of shares. We showed that it was less relevant than the complete hybrid strategy. This can be seen in Figure 3.1. The three ratios are high for the 25% highest Gammas with the notable exception of the 5% highest Gammas.

Thirdly, we wanted to illustrate the relevant limit for Gamma so as to make such a strategy relevant. Considering that interesting convertible bonds for this strategy are the bonds with the highest Gammas, we just have to find a threshold $\theta$ delimiting high Gammas and other Gammas. Figure 3.2 hence represents the three asymmetric performance ratios for groups of Gammas going from the centile $c$ to the maximum Gamma, for various $c$. As we considered wider ranges of Gammas than in Figure 3.1, we did not use a robust presentation of the ratios but simply the raw ratios of the group. Therefore, the right bound of the curves of Figure 3.2 (the ratios for the group of Gammas above the 99th centile) is different from the right bound of the solid lines of Figure 3.1 (a local median around the highest centile of Gammas), whereas they would have been identical if no median had been used in Figure 3.1. The left bound of the curves in Figure 3.2 indicates the average ratio on the whole sample. This can be appreciated as a lower bound of what high ratios are. Fortunately, the curves are almost always above their left bound, which confirms the relevance of investing in high Gammas. The choice of the threshold is also a problem related to the size of the investable universe. The tests have been done on a Euro universe represented by the index Exane Euro. As mentioned above, this index contained an average number of 36 convertible bonds in the period studied. Therefore, applying the strategy to only one convertible
bond in this universe is equivalent to choosing a threshold $\theta$ equal to 97.2%. This threshold leads to high ratios, as can be seen in Table 3.1. Applying the strategy to the equivalent of two convertible bonds provides high ratios too. Going further allows keeping high Sortino and upside-potential ratios, but the Omega ratio decreases more rapidly and the values obtained are close to the average ratio on the whole sample. Therefore, applying the strategy on the bonds with the two highest Gammas of the Euro universe seems to be a reasonable choice. Extending the universe to make it global would in addition make it possible to apply the hybrid strategy to more convertible bonds and therefore would increase the performance opportunities.

### Table 3.1: Performance ratios for various thresholds

<table>
<thead>
<tr>
<th>#Convertible bonds in Exane Euro</th>
<th>Threshold</th>
<th>U ratio</th>
<th>S ratio</th>
<th>Omega-0 ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>97.2%</td>
<td>0.579</td>
<td>0.091</td>
<td>1.187</td>
</tr>
<tr>
<td>2</td>
<td>94.4%</td>
<td>0.539</td>
<td>0.067</td>
<td>1.143</td>
</tr>
<tr>
<td>3</td>
<td>91.7%</td>
<td>0.448</td>
<td>0.030</td>
<td>1.071</td>
</tr>
<tr>
<td>4</td>
<td>88.9%</td>
<td>0.435</td>
<td>0.032</td>
<td>1.078</td>
</tr>
<tr>
<td>5</td>
<td>86.1%</td>
<td>0.432</td>
<td>0.045</td>
<td>1.116</td>
</tr>
<tr>
<td>36</td>
<td>0.0%</td>
<td>0.260</td>
<td>0.015</td>
<td>1.059</td>
</tr>
</tbody>
</table>

Finally, we want to see how such a strategy evolves over time, that is for various horizons. Indeed, all the previous tests were done over a one-day horizon. Such a short horizon can incur high transaction costs. It is hence useful to check that this strategy can also be applied to longer horizons. We show in Figure 3.3 the Omega-0 performance ratio depending both on the threshold $\theta$ and on the horizon. We see that the average ratio over all the convertible bonds, whatever the Gamma ($\theta = 0$), increases with the horizon, almost linearly. If we put the threshold so as to select only the convertible bond with the highest Gamma, the Omega-0 ratio (as well as the two other ratios, not represented here) increases.
up to a horizon of 20 business days. All the ratios then decline until they reach the average value of the whole sample after 40 business days of this strategy. This means that the strategy applied to this sole convertible bond of the Exane Euro index is interesting for horizons lower than 40 days. If we choose the two convertible bonds with the highest Gammas, the strategy is interesting for up to 10 business days, according to Sortino and Omega ratios (but for a longer horizon according to the upside-potential ratio).

![Figure 3.3: Omega-0 ratio for various horizons between 1 and 40 days](Source: Exane, Bloomberg, Natixis Asset Management)

Each curve corresponds to a particular threshold \( \theta \). The dotted line is the average ratio over the entire sample. The five other curves correspond to 1 to 5 bonds on average with the highest Gamma in the Exane Euro index. When the dotted line crosses any other curve, the corresponding split of the universe between high and low Gammas does not lead to asymmetric risk-adjusted profits above the average.

The decay of the performance across the horizon can be explained by the fact that the strategy calibration (the choice of the weight of the share) is made only once, at the initiation of the strategy. However, the delta as well as the Gamma evolves with the time and the greater the horizon, the less accurate the delta hedging. Dynamic hedging would allow better performances but transaction costs may become prohibitive. Therefore, rebalancing the strategy for the convertible bond with the highest Gamma every 20 days, as shown in Figure 3.3, seems reasonable.

To sum up, the hybrid strategy, with full delta hedging, leads to attractive asymmetric performances. Implementing this strategy for horizons lower than 40 days is relevant if the convertible bond has the highest Gamma of the European index. Lower Gammas would better suit shorter investment horizons.

2. Exploiting views on the underlying: trend and volatility

In addition to systematic strategies such that the hybrid strategy presented above, certain strategies are based on the views of portfolio managers. For example, if the portfolio manager anticipates a decline of the stock market, he will invest in convertible bonds with a low delta, so as to take advantage of the bond floor. On the contrary, if he expects the stock market to go up, higher deltas will provide higher expected gains.

Views can be related to economic outlooks as well as to the long-memory feature of the time series described in Section I.A.1. Moreover, portfolio managers can capitalize on views other than views on the direction of the stock market.
Convertible bonds can indeed be used to materialize views on the volatility of the stock market. This can be achieved in different ways.

Assuming that the portfolio manager anticipates a high volatility of the underlying share, he should buy one convertible bond and sell \( K/2 \) shares, where \( K \) is the conversion ratio. By doing so, the payoff is almost symmetric, whatever the direction of the evolution of the share price. The higher the amplitude of this evolution, the higher the gain of the strategy. However, credit risk is still a possible source of loss. Figure 3.4 shows the performance of such a strategy for a one-month horizon\(^\text{14}\) and a standard five-year convertible bond without any other option, calculated as the logarithmic ratio of the cash generated versus the cash invested. For example, if the bond is priced in units, it is equal to:

\[
\ln \left( \frac{P_{t+1} - S_{t+1} K / 2}{P_t - S K / 2} \right) \quad \text{eq. 11}
\]

where \( P \) is the price of the convertible bond and \( S \) the price of the underlying share. Contrary to the paragraph about the hybrid strategy, we used this standard performance measure because we did not observe any amount of cash invested close to zero, whatever the initial share price. The curves shown in Figure 3.4 indicate that the relevance of such a strategy strongly depends on the initial share price. In the present example, an initial share price of 85 leads to positive gains whatever the evolution of the share, to the notable exclusion of a dramatic decrease corresponding to a credit event. However, for another initial share price such as 50, the strategy is profitable if the share price decreases down to a value of 10 or increases up to a value above 130. As such, it is not really a volatility strategy. Indeed, since it is in the middle of the bond floor, the very low delta and the quite stable convertible bond prices cannot balance the losses induced by the short position on a share that is going up. A symmetric analysis can be made for the third case displayed, corresponding to an initial price of 150. The mixed profile of the convertible bond, with a delta not far from 50%, thus seems to be the better bet.\(^\text{15}\)

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14. This strategy depends all the more on the horizon that the pricing function of the convertible bond evolves over time. The figures provided in the rest of the paragraph must thus not be considered as anything other than an example.

15. The initial delta for a stock price of 85 is 39% in our example. 50% of delta can be obtained with a stock price of 95, which also provides excellent performances for this strategy.
Moreover, we wanted to highlight that the return curves, as shown in Figure 3.4, can all be mapped together by a translation. Indeed, for two pairs of initial prices, \((S_t, P_t)\) and \((S'_t, P'_t)\), hence corresponding to two different curves, we have the following equation, which illustrates the translation:

\[ r(S_t, S'_t) - r(S'_t, S_t) = \ln \left( \frac{P'_t - S'_t K / 2}{P_t - S_t K / 2} \right). \tag{12} \]

Figure 3.5 shows the value of this translation parameter. It indicates that the closer to 85 the initial share price, the more relevant this strategy as a volatility strategy. An initial share price of 90 could still be interesting but it must not be far from the target of 85.

An initial share price of 150 yields a value of -16%. Therefore, the performance curve for an initial share price of 150 is 16% below the ideal performance curve corresponding to an initial share price of 85.

An expected high volatility can also lead to a similar strategy, which consists in buying a convertible bond and \(K\) put options on the underlying share. Except if the initial share price is 85, it should allow an easier implementation of the view, with a strike chosen to be consistent with the forecasted volatility.

Finally, if the portfolio manager anticipates a low volatility, he should take the opposite positions to those set out above. Symmetric remarks regarding the initial share price must also be taken into account.

**B. LONG-TERM PERFORMANCE FACTORS**

Beyond the strategies specific to convertible bonds, it is instructive to analyze what drives the performance of funds of convertible bonds.

The long-term performance of convertible bonds is mainly driven by standard stock factors and bond factors. Ammann, Kind and Seiz (2010) addressed this issue for US convertible bond funds between 1985 and 2004. These funds are not only invested in convertibles. Dutordoir and Van de Gucht (2004) underlined that the part of such funds which is not invested in convertible bonds is mainly invested in stocks in the US, whereas European investors prefer bonds, as explained in Section I.B.1. Ammann, Kind and Seiz (2010) thus tested the significance of different kinds of factors:
Stock-related factors: equity market performance, difference of return between small and large-capitalization stocks, between high and low-book-to-market stocks, between high and low-past-return stocks.

Bond-related factors: bond market performance (Govies and Credit), term structure changes, credit spread, high-yield market performance.

Option-related factors: equity implied volatility, positive performance of the equity market above the bond market, equity market performance squared, convertible bond market.

The dependence on both stock and bond factors is linear and corresponds to the intuition that convertibles stand between a stock and a bond. Moreover, a specific class of mostly non-linear factors is added so as to reflect particularities of convertible bonds which could be considered as an option. This study focuses on the American market, so that it is stock-centered. All the stock factors are significant for explaining convertible bond performance. For bond-related and option-related factors, considered as additional factors, only the credit spread, the high-yield performance and the convertible bond market performance are significant, for an adjusted $R^2$ reaching 72%.

As we mentioned, the American market is stock-oriented, whereas the European market is bond-oriented. Vanpée and Van Campenhout (2013) confirm this dissimilarity on a sample ranging from 2000 to 2011. European convertible bond funds are more closely correlated to bond factors than are American funds. Asian funds behave more or less like European funds.

Vanpée and Van Campenhout (2013) showed that funds of global convertible bonds also depend on different factors, whether they are domiciled in the US or in Europe. Unsurprisingly, American portfolio managers depend more on stock-related factors than on bond-related factors. This is the opposite for European and Asian portfolio managers. This bias is due to the closeness between the convertible market and the equity market in the US, whereas it is closer to the bond market in Europe and in Asia.
CONCLUSION

In this paper, convertible bonds have been presented as a rich asset class per se and not only as a basic mix between equity and credit. This asset class is quite able to diversify a credit portfolio for investment horizons shorter than a month. It also currently relies on a long-memory characteristic, which makes long-term predictions more accurate and view-based strategies more profitable. Systematic strategies, such as investing on the convexity, can be rewarding for monthly horizons as well. New opportunities can also be offered by a global universe of convertible bonds. We analyzed the diversity of convertible bonds in the US, in Europe, in Japan or in the rest of Asia. This diversity increases the opportunities for strategic investment and for diversification for a portfolio manager.
A. HURST EXPONENT

The Hurst exponent, named after Harold Hurst (1951), is a statistic belonging to (0,1) which depicts different features of time series:

- If $H = 1/2$, where $H$ is the Hurst exponent, then, the series is a Brownian motion. The increments are independent random Gaussian variables.
- If $H > 1/2$, the increments are positively correlated. Such a series smooths singularities: it is a fractional primitive integral of a Brownian motion.
- If $H < 1/2$, the increments are negatively correlated. This creates singularities: it is the fractional derivative of a Brownian motion.

We thus see that this statistic is linked to the fractional Brownian motion, which has been introduced as such by Mandelbrot and Van Ness (1968), following works initiated by Kolmogorov (1940). It is a generalization of the standard Brownian motion, characterized by the equations:

\[
\begin{align*}
B_H(0) &= 0 \\
E\{B_H(t)\} &= 0 \\
E\{(B_H(t) - B_H(s))^2\} &= \sigma^2 |t - s|^{2H},
\end{align*}
\]

for times $t$ and $s$ and a volatility parameter $\sigma$. Properties such as the self-similarity of the process follow. What is interesting in our purpose is the long-term autocorrelation of increments:

\[
\text{Correl} \left( B_H(n+1) - B_H(n), B_H(1) - B_H(0) \right) = \frac{H(2H - 1)}{n^{3H - H}} + \sigma^2 \left( \frac{1}{n^{H - 1/2}} \right),
\]

from which we conclude that:

- the lower $H$ is, the faster the autocorrelation decreases when $n$ grows,
- for $H > 1/2$, the greater $H$, the greater the autocorrelation,
- long-term dependence is characterized by $H > 1/2$, since the sum of the autocorrelations for $n$ in a range $[1, N]$ is equivalent to $HN^{2H - 1/2}$, when $N$ tends toward infinity: this sum is infinite only for $H > 1/2$.

Another way of interpreting fractional Brownian motion, related to the fractional primitive and derivative we mentioned, consists in seeing it as the moving average of the increments of a standard Brownian motion, in which the weight is $(t - s)^{H - 1/2}$, for $t$ the current time and $s$ the time of the past increment considered. Therefore, if $H$ is equal to $1/2$, the weights are equal whatever the delay. If $H$ is greater than $1/2$, then the weights increase with the delay, whereas they decrease if $H < 1/2$. In this last case, the memory is destroyed and the series is very erratic. As can be seen in Figure A.1, the greater $H$, the smoother the time series and thus the easier any forecast.
The standard geometric Brownian motion is a dynamic which is broadly used for modelling the random evolution of the price of financial assets. The fact that it is a very easy tool has fostered such a success. It is defined by the following equation, in which $S$ is the asset price and $\mu$ its trend:

$$S_t = S_0 \exp\left(\mu t + \sigma B_{1/2}(t) - \frac{\sigma^2}{2} t\right). \quad \text{eq. 15}$$

It is so deeply ingrained in the mind of practitioners, that everybody implicitly uses it when prosaically annualizing a volatility with the $\sqrt{T}$-rule for example, where $T$ is the number of observations in one year for the chosen scale. However, it is no surprise that such an annualization rule is far from reality. The geometric fractional Brownian motion offers a way to annualize volatilities more properly and, more generally, describes how the dynamics of an asset price are more likely to behave. It is defined by:

$$S_t = S_0 \exp\left(\mu t + \sigma B_H(t) - \frac{\sigma^2}{2} t^{2H}\right). \quad \text{eq. 16}$$

The Hurst exponent can therefore be estimated so as to match the logarithmic returns at various scales. In other words, we are looking for the exponent $H$ such that the volatility should be annualized by a $T^H$-rule. This is the method used in this paper. Alternative estimation methods are developed by Mielniczuk and Wójtowylo (2007). The confidence interval for an estimated Hurst exponent $\hat{H}$ can be estimated by a bootstrap: bounds are given by quantiles of the empirical distribution of estimated Hurst exponents provided by a Monte Carlo of a simulated fractional Brownian motion of Hurst exponent $\hat{H}$, as suggested by Hall, Härdle, Kleinow and Schmidt (2000) and by Weron (2002). Figure A.1 shows the exponential of fractional Brownian motions for various Hurst exponents and with the same series of pseudo-random variables.

The time $t$ is in business days, the parameter $\sigma$ is $20\%/\sqrt{256}$, which corresponds to an annual volatility of $20\%$ in the case of $H = 1/2$. 

![Figure A.1: Exponential of fractional Brownian motions for various Hurst exponents and with the same series of pseudo-random variables](image)

Source: Natixis Asset Management
A.2 shows such confidence intervals depending on the value of the Hurst exponent.

![Figure A.2: Approximate bounds of a 90%-level confidence interval of Hurst exponents](source: Natixis Asset Management)

For a 300-day sliding window (purple) and a 100-day sliding window (light purple).

The Hurst exponent may be non-uniform. In this case, $H$ depends on time and one faces a multifractal Brownian motion as well as its geometric version for modelling asset price. Therefore, we estimate a dynamic $H$ on rolling windows. An alternative method consists in obtaining a kind of approximate distribution of the Hurst exponents. It is provided by the so-called singularity spectrum, introduced by Frisch and Parisi (1985) and which can be estimated by wavelets as exposed by Arneodo, Bacry, Jaffard and Muzy (1998). Instead, we focus on standard empirical distributions on Hurst exponents estimated on rolling windows.

**B. MULTiresolution CORRELATION**

The Hurst exponent is only one way of linking the statistical properties of a time series considered at different samplings. Another way to have a multiresolution analysis of the risk consists in representing the correlation of two time series at different time scales. This can be done with wavelets.

Wavelet techniques aim at decomposing a time series at different time scales. They thus have the same goal as the Fourier analysis. However, wavelets make it possible to more accurately depict the local specificities of a time series. In a sense, they are a kind of windowed Fourier decomposition with an adaptable window, that is to say a window whose size diminishes with the scale. Such a decomposition can have many applications in finance: making more reliable predictions by eliminating a noise (Garcin and Guégan (2016)) or by combining it with neural networks (Chauhan, Ravi and Chandra (2009)), describing time series according to different time scales to detect cycles, regime changes or any particular multiresolution structure. Theoretical and practical details on wavelets can be found in the book written by Mallat (2000) and numerous applications to finance in the thesis by Ranta (2010).

We are given a functional basis, $(\psi_{j,k})$, the wavelets, which are derived from a mother wavelet $\Psi \in L^2(\mathbb{R})$ by dilatations and translations:

$$\psi_{j,k} : t \in \mathbb{R} \mapsto 2^{j/2} \Psi(2^j t - k).$$

where $j$ and $k$, are respectively the resolution and the translation parameters and are integers in this discrete wavelet framework.
For a time series $y$, the empirical wavelet coefficient for parameters $j$ and $k$ is:

$$\langle y, \psi_{j,k} \rangle = \sum_{t=0}^{T} y(t) \psi_{j,k}(t). \quad \text{eq. 18}$$

The scaling coefficient is obtained by using the same formulas when replacing the mother wavelet $\Psi$ (derived in $\psi_{j,k}$) by the father wavelet, or scaling function, $\Phi$ (derived in $\phi_{j,k}$), which is uniquely defined for a given mother wavelet. In this paper, we use Daubechies mother wavelets with four vanishing moments. Moreover, we use stationary wavelets $^{16}$ in order to have invariance by translation and we truncate the coefficient series so as to eliminate singularities at the borders.

In this paper, we focus on multiresolution correlations. Interactions between time series are indeed supposed to depend on the time scale. For example, Epps (1979) has empirically revealed that correlation is expected to diminish with the time scale. He gives the following explanation: time series can have lagged interactions at a high frequency (and therefore low non-lagged correlations), whose effect is invisible at lower frequencies. From then, other justifications of the Epps effect have been found, such as: realized correlation estimators are biased when prices are not synchronized, as shown by Hayashi and Yoshida (2005), or the continuous underlying process is rounded by the tick discrete world, what implies another biases contributing to the Epps effect, as shown by Münnix, Schäfer and Guhr (2010).

The multiresolution correlation (examples provided by Gençay, Selçuk and Whitcher (2001)) is calculated using a multiresolution analysis, that is to say the decomposition of time series in approximation and details. Details are given by wavelet coefficients whereas approximation is given by the corresponding scale coefficients:

$$y(t) = \sum_{k} \langle y, \phi_{j,k} \rangle \phi_{j,k}(t) + \sum_{k} \langle y, \psi_{j,k} \rangle \psi_{j,k}(t), \quad \text{eq. 19}$$

for a given resolution $j$, which indicates the desired level of detail. Instead of calculating the correlation of the approximation terms of two series, we calculate the equivalent correlation of the corresponding scaling coefficients. Doing this for various resolution parameters makes it possible to draw the multiresolution correlation curves. In this paper, $y$ is the daily log-return and we also exclude at each scale the observations in the cone of influence of the borders.

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16. Details about stationary wavelets can be found in Nason and Silverman (1995) and in Pesquet, Krim and Carfantan (1996).
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